*Journal of Sound and Vibration* (2000) **234**(2), 277–296
doi:10.1006/jsvi.1999.2873, available online at http://www.idealibrary.com on IDEAL®

# A MODIFIED APPROACH BASED ON INFLUENCE COEFFICIENT METHOD FOR BALANCING CRANK-SHAFTS

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(*Received* 21 *August* 1999, *and in final form* 6 *December* 1999)

The conventional balancing machines utilize two-plane separation for the determination of equivalent imbalances. However, the imbalance masses of a crank-shaft cannot be corrected at arbitrary locations of the balancing planes. This study has presented a modified method for balancing crank-shafts by using the soft-pedestal machines. The modified influence coefficient method for asymmetrical rotor-bearing systems has been applied to balance crank-shafts. Also, the decomposition method for irremovable masses has been replaced by an iteration method based on an influence coefficient approach. Furthermore, the validity and accuracy of the modified approach are verified in balancing practical crank-shafts.



## 1. INTRODUCTION

The influence coefficient balancing method uses known trial masses to determine experimentally the sensitivity of a rotor-bearing system. It subsequently calculates a set of discrete correction masses which minimizes whirl responses. In conventional procedures, a trial mass is applied first to one of the balancing planes and the rotor responses are measured, and this process is repeated for all other balancing planes. An influence coefficient matrix is then obtained from these data.

Early research in rotor balance was conducted first by Thearle [1], then by Baker [2]. Their method was essentially a two-plane, two-sensor, single-speed and exact-point influence coefficient balancing procedure. Goodman [3] extended the influence coefficient procedure to include the least-squares method for balancing flexible rotor-bearing systems. Although the method had been known and published, it was more an art than a science. It is important, however, to evaluate the method because it is widely used and gives satisfactory results. Lund and Tonneson [4] examined the validity and accuracy of the influence coefficient method and investigated the influence of various instruments on the accuracy of the experiments. Tessarzik *et al.* experimentally evaluated the balancing precision of the influence coefficient method by the exact-point speed procedure [5] and the least-squares procedure [6]. Linear programming techniques were employed by Pilkey and Bailey [7] for regulating the balance weight magnitudes and by Pilkey *et al.* [8] for locating optimal balancing planes through constraint equations. Other important reports on analytical and experimental investigations involving influence coefficient balancing were surveyed by Darlow [9].

In the conventional influence coefficient method, a single trial mass applied to each balancing plane with measured displacement is employed to calculate influence coefficients and unbalance distribution. Since the conventional method disregards the unequal properties of rotating parts and asymmetry of bearings, it cannot provide an equivalent imbalance distribution.

In the paper of Kang *et al.* [10], a formulation of influence coefficient matrices was derived from motion equations for asymmetrical rotors using complex co-ordinate representation and the finite element method. On the basis of their study, the influence coefficient approach can be verified analytically and proven that it is not an art but a science. From the theoretical developments, this study proposes modified influence coefficient procedures for balancing the asymmetric rotor-bearing systems. Three modifications are made: (a) two trial masses are applied at two different positions of each balancing plane, (b) forward precessions are utilized to determine the influence coefficients and the original imbalance distribution, and (c) multi-plane balancing procedures are proposed for rigid crank-shaft rotors. On the basis of the present modifications, the imbalance correction can be obtained.

The engine crank-shafts are typically asymmetrical rotors, which have the unequal properties in two principal directions. Conventional balancing machines for crank-shafts are of the hard-pedestal type, by which the imbalance forces are recorded by two load cells mounted on two pedestals. Since the measured forces of two bearing supports can be equivalent to the imbalance forces of two balancing planes, statically, the asymmetrical properties of crank-shaft need no regard. However, the counterbalances of crank-shaft have provided multi-plane to balance, according to the exact point. The force-to-force method of the hard-pedestal type gives only two equivalent imbalances, but the crank-shaft with multi-plane can be corrected. Furthermore, balancing machines of the soft-pedestal type have an algorithm vibration sensitivity structure and multi-plane balancing. They provide more precise results of imbalance correction than balancing machines of the hard-pedestal type.

Some of the earliest general reference works to include discussions of balancing machines were those by Jeffcott [11], Timoshenko [12], and Kroon [13]. Similar discussions were presented in the Shock and Vibration Handbook [14], in which several machines and methods for balancing rigid rotors are described. Also, the International Standards Organization (ISO) has issued documentation on balancing machines and plane separation of two-plane rotors [15].

A two-plane, two-sensor, single speed and exact-point influence coefficient method is generally valid and utilizable for balancing the rigid rotor on a balancing machine either of a soft or a hard-type. Den Hartog [16] described two-plane separation utilized by a cradle-balancing machine for rigid rotors. He also described how the influence coefficient approach is used in Gisholt–Westinghouse balancing machines. Rao [17] gave both descriptions about the plane separation and influence coefficient approach individually for balancing the rigid rotors with two bearing planes. Actually, the plane-separation and influence coefficient approach are equivalent and similar, both being based on the linear theory of sensitivity.

In the operation of balancing machines of the soft-pedestal type, the principles of plane separation are utilized to separate imbalance effects into discrete planes. The modern balancing machine accomplishes this plane separation of imbalance by means of electrical networks. The rotor is supported by two bearings and vibration sensors at two points that convert response to voltage. Because of the opposite direction of conduction, the voltages and, by means of a voltage amplifier or divider, the imbalance effect due to that of the plane can be reduced to zero at another sensor. By similar reasoning, one of the sensors is read out, which is unaffected by the other plane, to indicate the imbalance on this plane.

Thus far, plane separation has been discussed in the above paragraphs for two-plane and rigid rotors only. Some rotors, such as crank-shafts, long turbine rotors, and generator rotors, etc., need balancing in several planes for reducing mass concentration at two planes. However, the commonly used balancing machines have some difficulty in balancing these complex rotors by using multi-plane correction. Kang *et al.* [18] derived an algorithm of plane separation based on the exact-point influence coefficient approach. From the analysis, a generalized procedure for multi-plane separation of balancing a rigid rotor is provided by an inference from two-plane separation and then three-plane separation. This process of multi-plane separation can be utilized by a balancing machine to correct a large number of planes simultaneously or successively.

Furthermore, Yeh and Yu [19] have provided theoretical description about the conventional method for balancing crank-shaft. They have utilized the two-plane and exact-point algorithm to determine imbalances on two calibrated counterbalancing planes. The imbalances have then been transferred to every counterbalance by the decomposition of vectors for locating removable masses at approximate positions. This method decomposes the vectors by using statics only, and thus the dynamical imbalance remains. Until now, the operations of crank-shaft balancing in industry have utilized a similar balancing method, but it does not provide a high-quality result.

Based on the balancing theory of the asymmetrical rotor and the principle of the multi-plane balancing rigid rotor on a balancing machine, this study proposes the modified method of soft-pedestal machine for balancing crank-shafts. The validity of the present formulation and the modified balancing method is verified both by computer simulations and by balancing experiments. All the examples indicate that the modified influence coefficient balancing method can yeild better balancing quality than the conventional balancing method does.

## 2. INFLUENCE COEFFICIENT METHODS OF CRANK-SHAFT

The influence coefficient method is well developed and widely used in rotor dynamics. In the paper of Kang *et al.* [10], a formulation of influence coefficient matrices was derived from motion equation for asymmetrical rotors using complex co-ordinate representation and the finite element method. Also, from the analysis, the modified influence coefficients are found correlated to forward precession and imbalance forces when the asymmetry of the bearings is considered. Due to the unequal properties in two principle directions, the present formulation results in two sets of modified influence coefficients. The formulation indicates that two trial masses in different directions are required in the two trial operations for each balancing plane.

On the basis of Kang *et al.* [10], the two influence coefficient matrices for a crank-shaft can be obtained

$$\sum_{j=1}^{J} (A_{kj}P_j + B_{kj}Q_j) = r_k, \quad k = 1, 2, \ldots, K, \tag{1}$$

where $J$ is the number of balancing planes and $K$ is the number of measured points. When a trial mass is applied on the $j$th balancing plane, the $k$th measured response $r_k$ is related to the balancing forces $P_j$ and $Q_j$ by the influence coefficients $A_{kj}$ and $B_{kj}$, respectively. The measured response of the $k$th point due to original imbalance is

$$r_k^o = \sum_{j=1}^{2} (A_{kj}P_j^0 + B_{kj}Q_j^0) = g_k^o + \mathrm{i}\gamma_k^o, \tag{2}$$

where $g_k^0$ and $\gamma_k^0$ are the real and the imaginary parts of $r_k^0$, respectively.

As a trial mass $P'_j + \mathrm{i}Q'_j$ is applied to the $j$th plane, equation (2) can be rewritten as

$$r'_k = \sum_{j=1}^{2} [A_{kj}(P_j^0 + P'_j) + B_{kj}(Q_j^0 + Q'_j)] = g'_k + \mathrm{i}\gamma'_k. \tag{3}$$

Similarly, another trial mass $P''_j + \mathrm{i}Q''_j$ is applied at different angular positions of the same $j$th balancing plane. Equation (3) can then be rewritten as

$$r''_k = \sum_{j=1}^{2} [A_{kj}(P_j^0 + P''_j) + B_{kj}(Q_j^0 + Q''_j)] = g''_k + \mathrm{i}\gamma''_k. \tag{4}$$

Subtracting equation (2) from equations (3) and (4), the following equations are obtained:

$$A_{kj}P'_j + B_{kj}Q'_j = r'_k - r_k^0, \qquad A_{kj}P''_j + B_{kj}Q''_j = r''_k - r_k^0. \tag{5}$$

The non-homogenous equation (5) can be solved for the modified influence coefficients that relate the response at the $k$th point to the imbalance force at the $j$th plane.

## 3. MEASURED RESPONSE OF SOFT-TYPE BALANCING MACHINE

For a soft-type balancing machine, the horizontal stiffness of its bearings is much smaller than the vertical stiffness. Thus, the vertical displacement is disregarded, and required measurements are only for lateral vibration.

For non-axisymmetrical bearings, the synchronous whirl follows an elliptical orbit which includes forward and backward precessions. The synchronous whirl excited by imbalance can be expressed as

$$q = q^+ \mathrm{e}^{\mathrm{i}\Omega t} + q^- \mathrm{e}^{-\mathrm{i}\Omega t}. \tag{6}$$

where $q^+$ and $q^-$ are forward and backward precessions, respectively, as shown in Figure 1(a). The rotating vector of the synchronous whirl is combined with two vectors. These two vectors rotate at the same speed $\Omega$, where one rotates in the positive $\Omega$ direction and the other in the opposite direction. Accordingly, the precessions $q^+$ and $q^-$ of the $k$th point can be expressed as

$$q_k^+ = \frac{1}{2}(y_c + z_s) + \frac{\mathrm{i}}{2}(z_c - y_s) = f_k \mathrm{e}^{-\mathrm{i}\theta}, \tag{7a}$$

where $f_k$ is the magnitude of forward precession; similarly

$$q_k^- = \frac{1}{2}(y_c - z_s) + \frac{\mathrm{i}}{2}(z_c + y_s). \tag{7b}$$

For a balancing machine of the soft-pedestal type, the rotor oscillates in a horizontal direction and a photodetector on a keyphasor is installed to detect the reflective mark or keyway for each revolution as shown in Figure 1(b). When the rotor rotates in a counterclockwise direction,

$$y = y_c \cos \Omega t + y_s \sin \Omega t \tag{8}$$

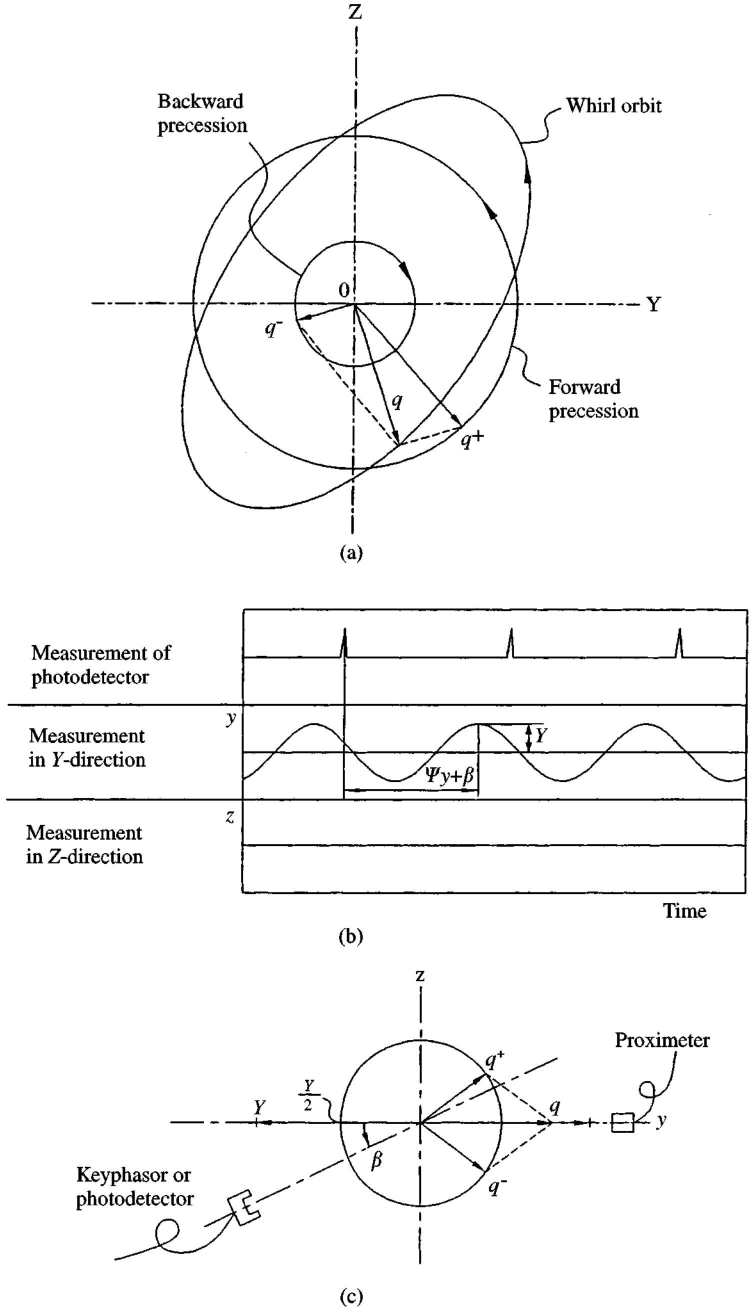


Figure 1. Whirl orbit and measurements of unbalance responses: (a) whirl orbit of a rotor mounted on anisotropic bearings; (b) typical measurements of balancing machine with soft pedestals; (c) whirl orbit of a rotor mounted on soft pedestals.

and

$$z \cong 0 \tag{9}$$

can be measured, where $y_c = y\cos(\Psi_y + \beta)$, $y_s = \sin(\Psi_y + \beta)$, and the phase angle $\Psi_y$ is determined by the difference between two measurements of the photodetector and proximeter. Equations (7a) and (7b) can be reduced to

$$q^{+} = \frac{1}{2}(y_c - \mathrm{i}y_s) \tag{10a}$$

and

$$q^{-} = \frac{1}{2}(y_c + \mathrm{i}y_s). \tag{10b}$$

Thus, the amplitudes of both forward and backward precessions are half of the maximum response as shown in Figure 1(c). The amplitude of forward precession is obtained by the determination of

$$f_k = |q^{+}| = Y/2, \tag{11}$$

where $Y$ is the amplitude of horizontal displacement. When a trial mass is applied to one of one balancing planes and the rotor responses are measured, the amplitude of forward precession is half of the maximum amplitude of horizontal vibration.

Thus, the influence coefficient matrices of unsymmetrical rotors with non-axisymmetrical bearings can be determined by the forward precession of the synchronous whirl and the trial mass.

## 4. DETERMINATION OF IMBALANCE

Rearranging equation (2) to real and imaginary parts and assembling into a matrix form results in

$$\begin{aligned} ([A^R] + [B^R])\{P_v^0\} + (-[A^I] + [B^I])\{P_w^0\} &= \{g^0\}, \\ ([A^R] + [B^R])\{P_v^0\} + ([A^I] - [B^I])\{P_w^0\} &= \{\gamma^0\}, \end{aligned} \tag{12}$$

where $[A^R] + \mathrm{i}[A^I] = [A]$ and $[B^R] + \mathrm{i}[B^I] = [B]$. Equation (12) can be assembled into matrix form:

$$[C]\{\mu\} = \{r\}, \tag{13}$$

where $\{\mu\} = (\{P_v^0\}^{\mathrm{T}}\{P_w^0\}^{\mathrm{T}})^{\mathrm{T}}$, $\{r\} = (\{g^0\}^{\mathrm{T}}\{\gamma^0\}^{\mathrm{T}})^{\mathrm{T}}$, and the dimension of $[C]$ is $2K \times 2J$. All the elements of $[A^R]$, $[A^I]$, $[B^R]$ and $[B^I]$ have been obtained by solving equation (5).

Equation (13) can be rewritten as

$$[D]\{\mu\} = \{f\}, \tag{14}$$

where $\{f\}$ is the vector of amplitudes of forward precession, $\{f\} = \{r\}/2$ and $[D] = [C]/2$.

For rigid-body balance number of balancing planes and measurement locations, the solution approaches of equivalent imbalance distribution can be classified into the following three categories:

(1) In the case where the number of measurements is less than the number of balancing planes, the equivalent imbalance is obtained by pre-multiplying the measured response vector with the pseudo-inverse matrix of $[D]$:

$$\{\mu\} = [D]^{\mathrm{T}}([D][D]^{\mathrm{T}})^{-1}\{f\}. \tag{15}$$

The method cannot give a unique and credible solution. Thus, it is seldom utilized in industrial application.

(2) In the case where the number of measurements is greater than the number of balancing planes, no attempt is made in the balancing process to reduce the vibration entirely. However, the sum of squares of the displacement amplitudes can be minimized by using the least-squares procedure. The following algorithm that utilizes weighting factors can be used. The solution of the imbalances is calculated by

$$\{\mu\} = ([D]^T[W][D])^{-1}[D]^T[W]\{f\}, \tag{16}$$

where $[W]$ is a diagonal matrix, which is composed of the weighting factors.

If the method leads to correction masses which are too heavy and difficult to apply, the magnitudes of the balancing masses are regulated through constraint equations by the iterative programming procedure, namely, the constrained optimal design technique.

(3) In the case where the number of measurements is equal to the number of balancing planes, the equivalent imbalance is obtained by pre-multiplying the measured response vector with the inverse matrix of $[D]$:

$$\{\mu\} = [\mathrm{D}]^{-1}\{f\}. \tag{17}$$

## 5. DECOMPOSITION METHOD FOR CORRECTION

The operation speeds of a crank-shaft are always far below their first mode frequency. Thus, the crank-shaft can be treated with the rigid balancing method. Only two measurements are needed in the balancing operation, and displacements of other positions can be determined by linear interpolation. Furthermore, two equivalent imbalances located on two balancing planes can be determined by using equation (17).

However, the configuration of a crank-shaft is non-symmetric about the spin axis, due to multi-counterbalancing planes. Thus, a crank-shaft cannot be corrected by removing masses at arbitrary locations of balancing planes. The equivalent imbalance must be decomposed into components on two axes of a co-ordinate divided by a specified angle $\phi$. As shown in Figure 2, the angle $\phi$ is measured from the right side to the left side of a counterbalance where the mass is removable.

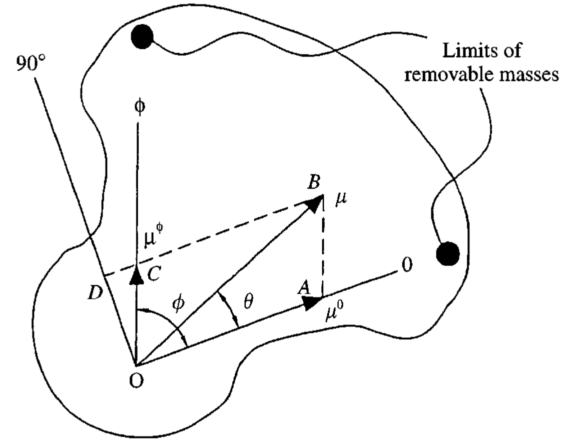


Figure 2. Angle co-ordinates of $\phi$.

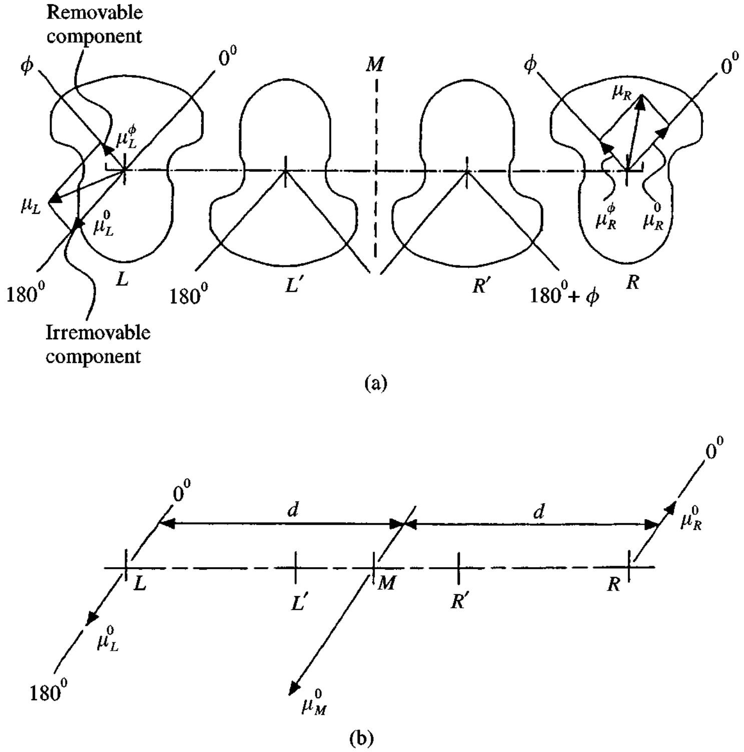


Figure 3. Vector decomposition of the conventional method.

The components of imbalance $\mu$ on both axes of the $\phi$-co-ordinate are obtained from both sides $\overline{OA}$ and $\overline{OC}$ of the parallelogram $OABC$. The decomposition of $\mu$ is shown as

$$\mu^0 = \overline{OA} = \mu \cos\theta - \mu \sin\theta \cot\phi,$$
$$\mu^\phi = \overline{OC} = \mu \sin\theta / \sin\phi, \tag{18}$$

where $\theta$ is the included angle of the vector of $\mu$ and 0-axis of the co-ordinate.

Balancing multi-plane crank-shafts by the rigid-rotor method does not allow the removal of masses from two counterbalances according to computing results. The irremovable mass on both planes should be decomposed to other counterbalances where the masses can be removed.

The conventional decomposition method of the irremovable imbalance uses the principle of force balance and moment balance. Take for example a four-cylinder crank-shaft as shown in Figure 3(a). The counterbalances are symmetricall about the mid point of the rotating axis. Firstly, this crank-shaft is balanced by using three planes $L$, $M$ and $R$. The plane $M$ is virtual and located at the mid point of the rotating axis. Both planes $L$ and $R$ are used to determine equivalent imbalances $\mu_L$ and $\mu_R$ by the two-plane method. Then these imbalances are decomposed into two components along the $\phi$-co-ordinate as $\mu_L^0$, $\mu_L^\phi$ and $\mu_R^0$, $\mu_R^\phi$. When these components are positive values, they can be directly removed from the $L$ or $R$ counterbalance. Otherwise, the component is negative; the irremovable mass at the $L$ plane has to be decomposed into two planes $M$ and $R$ by balancing forces and moments about point $L$.

As shown in Figure 3(b), $\mu_L^0$ is decomposed by the determinations

$$\mu_R^0 + \mu_M^0 = \mu_L^0 \tag{19}$$

due to the equilibrium of forces, and

$$2d\,\mu_R^0 + \mathrm{d}\,\mu_M^0 = 0 \tag{20}$$

due to the equilibrium of moments. Thus, the imbalance $\mu_L^0$ in plane $L$ can be removed by an identical amount of $\mu_M^0/2$ from both planes $L'$ and $R'$ in the location of the $180^\circ$ co-ordinate, and removed by $\mu_R^0$ from plane $R$ in the location of the $0^\circ$ co-ordinate.

Similarly, when the component of equivalent imbalance is on the irremovable position of the $R$ plane, the component of the imbalance component can be decomposed into the $L'$, $R'$ and $L$ planes by using a similar determination of equations (19) and (20).

## 6. INTERATION METHOD OF THE CORRECTION DETERMINATION

In this study, a modified method is proposed. This method utilizes total combinations of any two counterbalances to be correction planes. Many correction sets can be obtained from the determination of equivalent imbalances by using equation (17).

$$\{\mu\}_n = [D]_n^{-1}\{f\}, \quad n = 1, 2, \ldots, N, \tag{21}$$

where $N$ is the total amount of sets of corrections.

According to one of the sets of corrections, the removal of imbalance from the crank-shaft can improve its dynamic balance. The set of corrections which has the least amount of masses to be removed may be chosen as the initial balancing procedure. However, it is possible that there are irremovable components of imbalance residue at both correction planes; some further correction procedures are needed to remove these residual imbalances.

The imbalance responses of residual imbalances can be measured and substituted into equation (21). $N$ sets of the further corrections $\{\mu_r\}$ can be obtained from

$$\{\mu_r\}_n = [D]_n^{-1}\{f_r\}, \quad n = 1, 2, \ldots, N, \tag{22}$$

where $\{f_r\}$ represents the vector of responses due to residual imbalances.

Any one set of $\{\mu_r\}_n$ can be chosen to be the further correction for the removal of imbalance from the corresponding planes. Then the crank-shaft has to be further balanced. It is noted that the same set of correction planes may not be chosen, because the same correction planes give the same results of the residual imbalances that are not removable. If residual responses are still measured, the iterative procedures are to be performed until the crank-shaft is fully balanced or the measured responses due to residual imbalances are allowed.

## 7. COMPARISON BETWEEN BALANCING METHODS

The flow charts of the conventional method and the modified method for crank-shaft balancing are shown in Figure 4. For both methods, two balancing planes and two measurement locations are chosen first and the modified influence coefficient matrices are determined by using two trial operations for each plane. At some balancing speed, two trial masses are applied at two different positions for each balancing plane. The influence coefficients are determined by using equation (5). Then, the equivalent imbalances can be

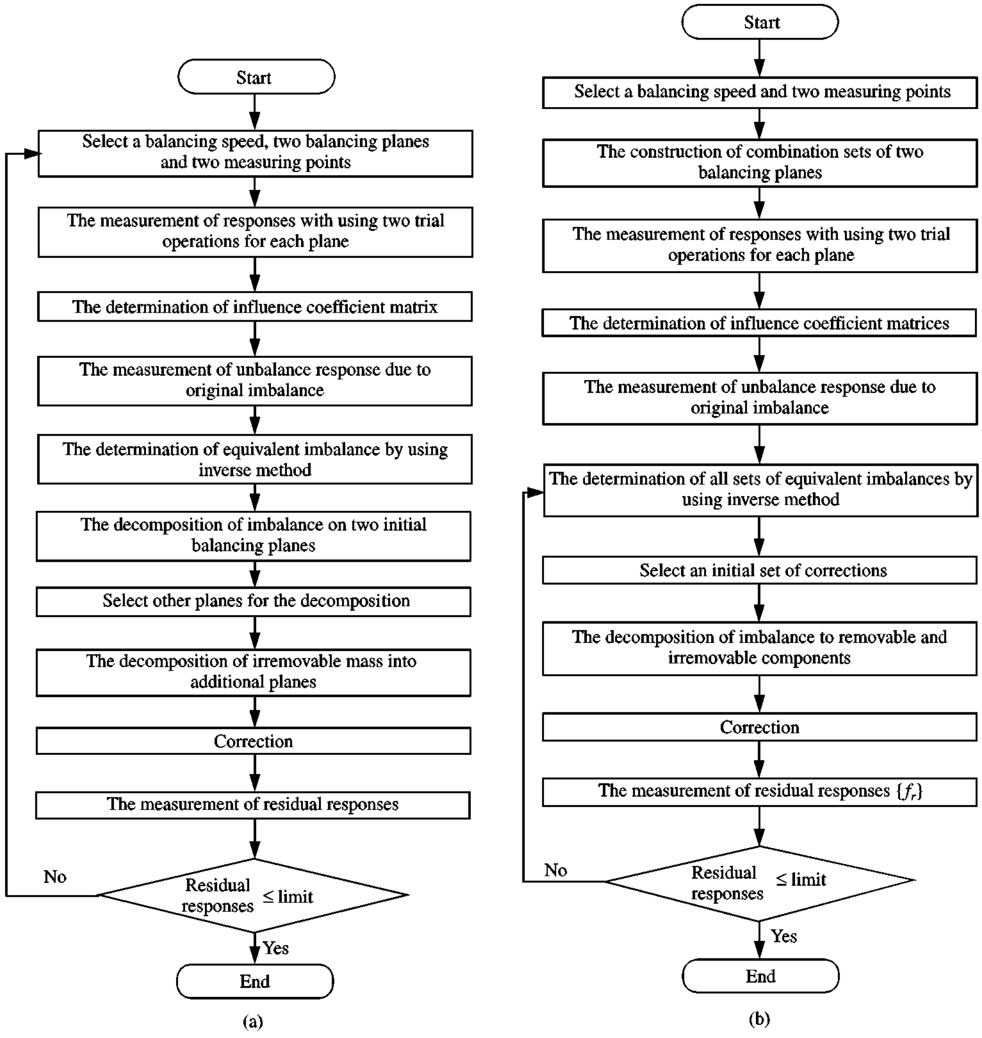


Figure 4. Flow charts for crank-shaft balancing: (a) the conventional method; (b) the modified method.

obtained by pre-multiplying the responses of the forward whirl due to original imbalance by the inverse of the influence coefficient matrix.

In the conventional method, the determination of equivalent imbalances utilizes two counterbalances at both ends of the crank-shaft. Then the decomposition of irremovable mass to other counterbalances is according to force and moment equilibrium. This method is also utilized in hard pedestal balancing machines. In this type of machine, the rotating forces due to imbalances by two load sensors and the equivalent imbalances are determined and the irremovable mass is decomposed into other counterbalances.

The balancing quality of the conventional method is determined by each and every procedure. Since the balancing speed and balancing planes are recalled for reselection when final residual response is not accepted, the quality is strongly affected by factors including the robustness of influence coefficients, the position and amount of trial mass, the sensitivity and accuracy of the balancing machine, the configuration of the crank-shaft, the measurement noise and the determination error.

In the modified method, all combinations of two arbitrary counterbalances have the responsibility of balancing planes. For the determination of equivalent imbalances, each set has itself an influence coefficient matrix with respect to identical measurement positions and equivalent imbalances. The initial correction planes are selected due to the least amount of equivalent imbalances and the largest removable components.

The irremovable imbalance $\{\mu_r\}$ induces the residual response $\{f_r\}$. Substituting $\{f_r\}$ into equation (21), new sets of correction masses are obtained. Also, the set with the least amount of correction masses is further chosen. If there is no complete set for residual imbalances in the removable range, the procedures can be kept until the residual responses are fully accepted.

The influence coefficient approach is utilized iteratively by the modified method. When the equivalent imbalance has an irremovable component, the determination procedure for residual imbalance on other counterbalances are repeated. Thus, it can continuously approach a higher balancing quality over time.

## 8. CASE STUDIES

Two crank-shafts are used to illustrate the validity of the proposed method.

(1) A four-cylinder, in-line crank-shaft as shown in Figure 5 is mounted on a balancing machine. The included angle between two limits of correction positions at each counterbalance is 95° approximately. The balancing data for the determination of influence coefficients and the measurements of imbalance responses are shown in Table 1. This table indicates that there are six sets of equivalent imbalances that can be obtained from the determination of equation (21). The imbalance masses can be decomposed into components on both axes of the 95°-co-ordinate as shown in Table 2. Each set of imbalances has at least one component that is irremovable mass components from the crank-shaft. For example, the third set of corrections, planes 1 and 4, are selected. The primary correction for it has the least amount of equivalent imbalances and the largest removable imbalance components. The equivalent imbalances on planes 1 and 4 have components on locations at 275 and 180°, respectively; the mass cannot be removed from these locations as shown in Figure 6.

By using the conventional method, residual imbalances due to irremovable mass are decomposed into other counterbalances. By using the modified method, six sets of equivalent residual imbalances are utilized. For the same residual imbalance, six sets

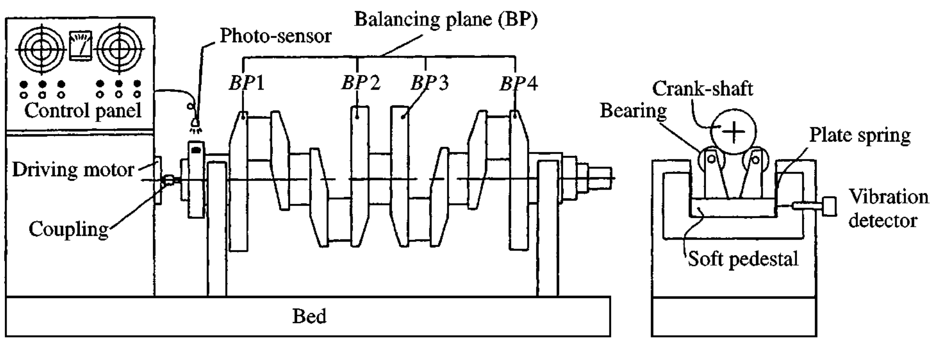


Figure 5. A four-cylinder in-line crank-shaft mounted on a soft-pedestal balancing machine.

TABLE 1

*Balancing data for the crank-shaft of an in-line four-cylinder engine*

Measurement data due to the modified method

| Measurement point | | Response (mm) 1 | Response (mm) 2 |
|---|---|---|---|
| Original imbalance | | $0{\cdot}2435\angle 85^\circ$ | $0{\cdot}2817\angle 244^\circ$ |
| The trial mass applied at plane 1 | The first position | $0{\cdot}2388\angle 120^\circ$ | $0{\cdot}2579\angle 250^\circ$ |
| | The second position | $0{\cdot}3868\angle 93^\circ$ | $0{\cdot}3056\angle 248^\circ$ |
| The trial mass applied at plane 2 | The first position | $0{\cdot}2340\angle 101^\circ$ | $0{\cdot}3247\angle 235^\circ$ |
| | The second position | $0{\cdot}3056\angle 90^\circ$ | $0{\cdot}2483\angle 235^\circ$ |
| The trial mass applied at plane 3 | The first position | $0{\cdot}2387\angle 91^\circ$ | $0{\cdot}3438\angle 230^\circ$ |
| | The second position | $0{\cdot}2722\angle 86^\circ$ | $0{\cdot}2292\angle 227^\circ$ |
| The trial mass applied at plane 4 | The first position | $0{\cdot}2770\angle 71^\circ$ | $0{\cdot}4202\angle 224^\circ$ |
| | The second position | $0{\cdot}1910\angle 78^\circ$ | $0{\cdot}1910\angle 210^\circ$ |

Product of mass and radius: 848·5 (g mm)

Influence coefficient ($g^{-1}$) due to the modified method

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $A_{11}$ | −0·0104 | $A_{21}$ | −0·0027 | $A_{31}$ | 0·0026 | $A_{41}$ | 0·0008 |
| $A_{12}$ | 0·0031 | $A_{22}$ | −0·0106 | $A_{32}$ | −0·0007 | $A_{42}$ | 0·0022 |
| $A_{13}$ | −0·0049 | $A_{23}$ | −0·0010 | $A_{33}$ | −0·0046 | $A_{43}$ | −0·0009 |
| $A_{14}$ | 0·0016 | $A_{24}$ | −0·0047 | $A_{34}$ | 0·0014 | $A_{44}$ | −0·0037 |
| $A_{15}$ | −0·0019 | $A_{25}$ | −0·0003 | $A_{35}$ | −0·0072 | $A_{45}$ | −0·0008 |
| $A_{16}$ | 0·0002 | $A_{26}$ | −0·0024 | $A_{36}$ | 0·0024 | $A_{46}$ | −0·0063 |
| $A_{17}$ | 0·0051 | $A_{27}$ | 0·0014 | $A_{37}$ | −0·0132 | $A_{47}$ | −0·0029 |
| $A_{18}$ | −0·0014 | $A_{28}$ | 0·0041 | $A_{38}$ | 0·0031 | $A_{48}$ | −0·0117 |

Measurement data due to the conventional method

| Measurement point | Response (mm) 1 | Response (mm) 2 |
|---|---|---|
| Original imbalance | $0{\cdot}2435\angle 85^\circ$ | $0{\cdot}2817\angle 244^\circ$ |
| The trial mass applied at plane 1 | $0{\cdot}2388\angle 120^\circ$ | $0{\cdot}2579\angle 250^\circ$ |
| The trial mass applied at plane 4 | $0{\cdot}2770\angle 71^\circ$ | $0{\cdot}4202\angle 224^\circ$ |

Product of mass and radius: 848·5 (g mm)

Influence coefficient ($g^{-1}$) due to the conventional method

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $A_{11}$ | −0·0104 | $A_{21}$ | −0·0027 | $A_{31}$ | 0·0026 | $A_{41}$ | 0·0008 |
| $A_{12}$ | 0·0027 | $A_{22}$ | −0·0104 | $A_{32}$ | −0·0008 | $A_{42}$ | 0·0026 |
| $A_{13}$ | 0·0051 | $A_{23}$ | 0·0014 | $A_{33}$ | −0·0132 | $A_{43}$ | −0·0029 |
| $A_{14}$ | −0·0014 | $A_{24}$ | 0·0051 | $A_{34}$ | 0·0029 | $A_{44}$ | −0·0132 |

can be utilized for further correction. One of the sets is chosen, which is shown in Figure 7. The final correction is attained because both residual imbalances on planes 2 and 4 are located in the range of removable mass. The balancing results of this crank-shaft using the conventional method and the modified method are shown in Table 3.

Table 2

*Equivalent imbalance of two arbitrary balancing planes in the first case*

| No. of balancing planes | | The left plane | The right plane |
|---|---|---|---|
| Left | Right | Imbalance (g mm/deg) | Imbalance (g mm/deg) |
| 1 | 2 | 1144·3 ∠ 0°<br>− 2237·6 ∠ 95° | 1706·7 ∠ 180°<br>− 2191·3 ∠ 275° |
| 1 | 3 | 708·3 ∠ 0°<br>− 1600·6 ∠ 95° | 1345·3 ∠ 180°<br>− 1661·2 ∠ 275° |
| 1 | 4 | 32·3 ∠ 0°<br>− 917·5 ∠ 95° | − 739·9 ∠ 0°<br>988·2 ∠ 95° |
| 2 | 3 | − 2926·9 ∠ 180°<br>5467·6 ∠ 275° | 3639·5 ∠ 180°<br>− 5716·8 ∠ 275° |
| 2 | 4 | − 132·5 ∠ 180°<br>1497 ∠ 275° | − 762·1 ∠ 0°<br>1638·8 ∠ 95° |
| 3 | 4 | − 36·7 ∠ 180°<br>2093·3 ∠ 275° | − 678·8 ∠ 0°<br>2299 ∠ 95° |

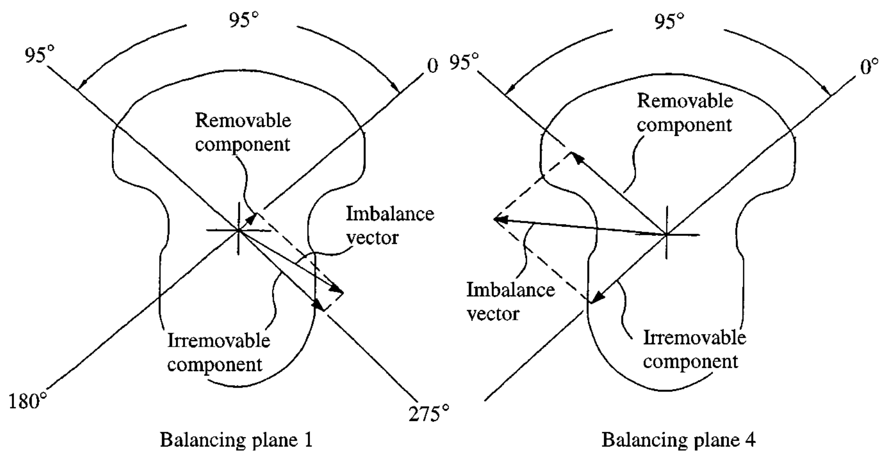


Figure 6. The third set of imbalance vectors in the first case.

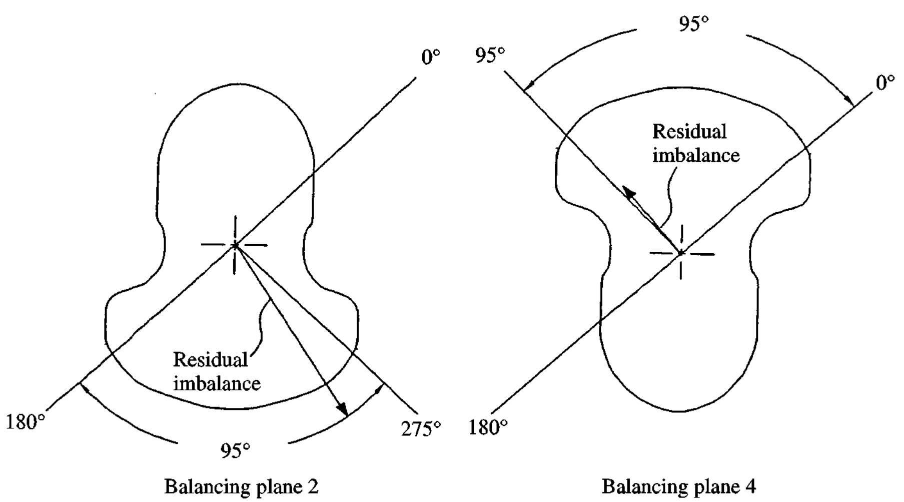


Figure 7. Removable mass of the final residual imbalance vector in the first case.

TABLE 3

*A comparison of correction between two methods in the first case*

| No. of balancing planes | The modified method | | The conventional method | |
|---|---|---|---|---|
| | Location angle (deg) | Imbalance (g mm) | Location angle (deg) | Imbalance (g mm) |
| 1 | 0 | 610·8 | 0 | 736·4 |
| | 95 | 0 | 95 | 0 |
| 2 | 180 | 101·4 | 180 | 736·4 |
| | 275 | 1475·4 | 275 | 924·2 |
| 3 | 180 | 1187·3 | 180 | 736·4 |
| | 275 | 116·1 | 275 | 924·2 |
| 4 | 0 | 0 | 0 | 0 |
| | 95 | 1639·1 | 95 | 1765·9 |
| Total amounts | — | 5130·1 | — | 5823·5 |

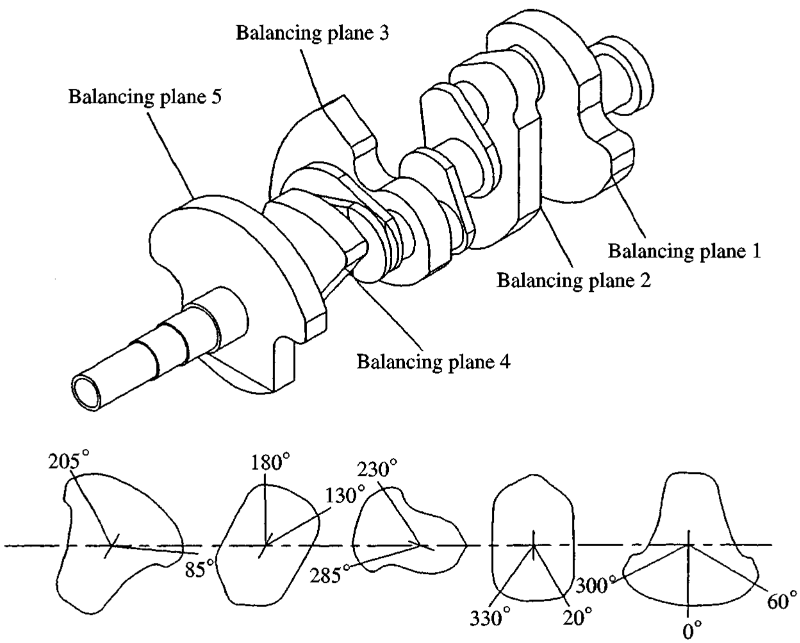


Figure 8. Crank-shaft of a V6-cylinder engine.

(2) The second case is a crank-shaft of a six-cylinder V-shaped engine as shown in Figure 8, which has five counterbalances. The included angles of removable regions for each counterbalance are not all the same. The balancing data for determination of the influence coefficient and the measurements of imbalance responses are shown in Table 4. There are 10 sets of equivalent imbalances that can be obtained from the determinations of equation (21). The imbalance masses can be decomposed into components by a co-ordinate of arbitrary angle $\phi$, as shown in Table 5. Also, Table 5 indicates that the fourth set is preferred to be corrected initially since it has the fewest equivalent imbalances and the largest components of removable imbalance mass.

TABLE 4

*Balancing data for the crank-shaft of a V6-cylinder engine*

Measurement data due to the modified method

| Measurement point | | Response (mm) 1 | Response (mm) 2 |
|---|---|---|---|
| Original imbalance | | $0{\cdot}3295\angle 275°$ | $0{\cdot}2006\angle 101°$ |
| The trial mass applied at plane 1 | The first position | $0{\cdot}3390\angle 242°$ | $0{\cdot}1910\angle 96°$ |
| | The second position | $0{\cdot}1576\angle 300°$ | $0{\cdot}1815\angle 105°$ |
| The trial mass applied at plane 2 | The first position | $0{\cdot}3295\angle 249°$ | $0{\cdot}2053\angle 107°$ |
| | The second position | $0{\cdot}1767\angle 279°$ | $0{\cdot}2244\angle 100°$ |
| The trial mass applied at plane 3 | The first position | $0{\cdot}3199\angle 264°$ | $0{\cdot}2674\angle 127°$ |
| | The second position | $0{\cdot}2769\angle 277°$ | $0{\cdot}3104\angle 95°$ |
| The trial mass applied at plane 4 | The first position | $0{\cdot}3438\angle 284°$ | $0{\cdot}3677\angle 141°$ |
| | The second position | $0{\cdot}3677\angle 274°$ | $0{\cdot}4154\angle 93°$ |
| The trial mass applied at plane 5 | The first position | $0{\cdot}3263\angle 289°$ | $0{\cdot}4202\angle 146°$ |
| | The second position | $0{\cdot}4154\angle 273°$ | $0{\cdot}4775\angle 91°$ |

Product of mass and radius:1380 (g mm)

Influence coefficient ($g^{-1}$) due to the modified method

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $A_{11}$ | – 0·0086 | $A_{21}$ | 0·0013 | $A_{31}$ | 0·0008 | $A_{41}$ | – 0·0003 |
| $A_{12}$ | – 0·0023 | $A_{22}$ | – 0·0087 | $A_{32}$ | 0·0004 | $A_{42}$ | 0·0010 |
| $A_{13}$ | – 0·0067 | $A_{23}$ | 0·0069 | $A_{33}$ | – 0·0010 | $A_{43}$ | 0·0000 |
| $A_{14}$ | 0·0000 | $A_{24}$ | – 0·0070 | $A_{34}$ | 0·0000 | $A_{44}$ | – 0·0011 |
| $A_{15}$ | – 0·0028 | $A_{25}$ | – 0·0005 | $A_{35}$ | 0·0056 | $A_{45}$ | – 0·0008 |
| $A_{16}$ | – 0·0002 | $A_{26}$ | – 0·0024 | $A_{36}$ | – 0·0005 | $A_{46}$ | – 0·0051 |
| $A_{17}$ | 0·0025 | $A_{27}$ | – 0·0002 | $A_{37}$ | – 0·0113 | $A_{47}$ | 0·0016 |
| $A_{18}$ | 0·0001 | $A_{28}$ | 0·0018 | $A_{38}$ | – 0·0008 | $A_{48}$ | – 0·0099 |
| $A_{19}$ | 0·0040 | $A_{20}$ | – 0·0005 | $A_{39}$ | – 0·0141 | $A_{49}$ | 0·0017 |
| $A_{110}$ | 0·0003 | $A_{210}$ | 0·0039 | $A_{310}$ | – 0·0014 | $A_{410}$ | – 0·0128 |

Measurement data due to the conventional method

| Measurement point | Response (mm) 1 | Response (mm) 2 |
|---|---|---|
| Original imbalance | $0{\cdot}3295\angle 275°$ | $0{\cdot}2006\angle 101°$ |
| The trial mass applied at plane 1 | $0{\cdot}3390\angle 242°$ | $0{\cdot}1910\angle 96°$ |
| The trial mass applied at plane 5 | $0{\cdot}3263\angle 289°$ | $0{\cdot}4202\angle 146°$ |

Product mass and radius:1380 (g mm)

Influence coefficient ($g^{-1}$) due to the conventional method

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $A_{11}$ | – 0·0086 | $A_{21}$ | 0·0013 | $A_{31}$ | 0·0008 | $A_{41}$ | – 0·0003 |
| $A_{12}$ | – 0·0013 | $A_{22}$ | – 0·0086 | $A_{32}$ | 0·0003 | $A_{42}$ | 0·0008 |
| $A_{13}$ | 0·0040 | $A_{23}$ | – 0·0005 | $A_{33}$ | – 0·0141 | $A_{43}$ | 0·0017 |
| $A_{14}$ | 0·0005 | $A_{24}$ | 0·0040 | $A_{34}$ | – 0·0017 | $A_{44}$ | – 0·0141 |

TABLE 5

*Equivalent imbalance of two arbitrary balancing planes in the second case*

| No. of balancing planes | | The left plane | The right plane |
|---|---|---|---|
| Left | Right | Imbalance (g mm/deg) | Imbalance (g mm/deg) |
| 1 | 2 | 6430·2 ∠ 300° | − 6908·8 ∠ 330° |
| | | − 830·8 ∠ 60° | 438·6 ∠ 20° |
| 1 | 3 | 2661·4 ∠ 300° | 157·1 ∠ 230° |
| | | − 470 ∠ 60° | − 1930·2 ∠ 285° |
| 1 | 4 | 1887·9 ∠ 300° | 1271·6 ∠ 130° |
| | | − 477·3 ∠ 60° | − 519·6 ∠ 180° |
| 1 | 5 | 1720·9 ∠ 300° | 934·6 ∠ 85° |
| | | − 481·5 ∠ 60° | 382·2 ∠ 205° |
| 2 | 3 | 5165·6 ∠ 330° | 0 ∠ 230° |
| | | − 4234 ∠ 20° | − 3267 ∠ 285° |
| 2 | 4 | 3198·1 ∠ 330° | 1923·3 ∠ 130° |
| | | − 2758·2 ∠ 20° | − 896·1 ∠ 180° |
| 2 | 5 | 2845·4 ∠ 330° | 1321 ∠ 85° |
| | | − 2495·4 ∠ 20° | 458·8 ∠ 205° |
| 3 | 4 | 811·7 ∠ 230° | 5206·5 ∠ 130° |
| | | 4949·7 ∠ 285° | − 2714·9 ∠ 180° |
| 3 | 5 | − 790·9 ∠ 230° | 2940·4 ∠ 85° |
| | | 4544·4 ∠ 285° | 835·3 ∠ 205° |
| 4 | 5 | − 16449 ∠ 130° | 12203 ∠ 85° |
| | | 8802·6 ∠ 180° | 3466·7 ∠ 205° |

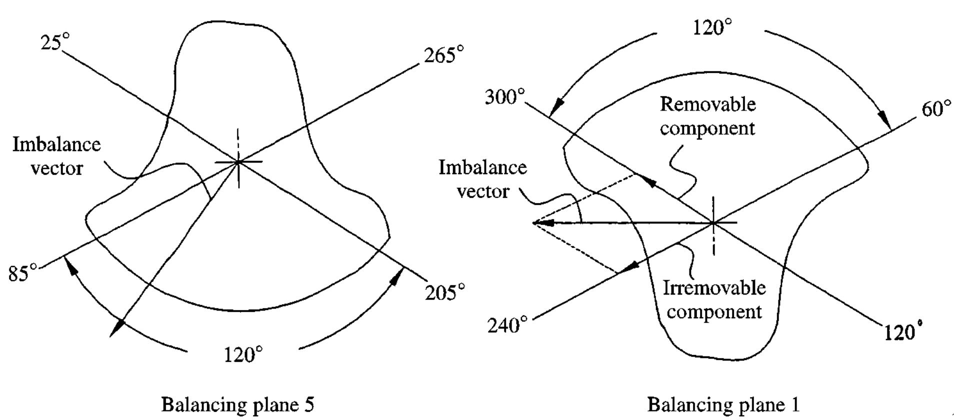


Figure 9. Imbalance vectors of the initial set for correction in the second case.

In this case, the equivalent imbalance on plane 1 has a component on the 240° location which cannot be corrected, as shown in Figure 9. By using the conventional method, the residual imbalances are decomposed to other correction planes by utilizing equations (19) and (20).

By using the modified method, 10 sets of residual imbalances are obtained from equation (21) iteratively. Finally, the residual imbalances have been reduced into the range of removable mass as shown in Figure 10. The balancing results of this crank-shaft due to both methods are shown in Table 6.

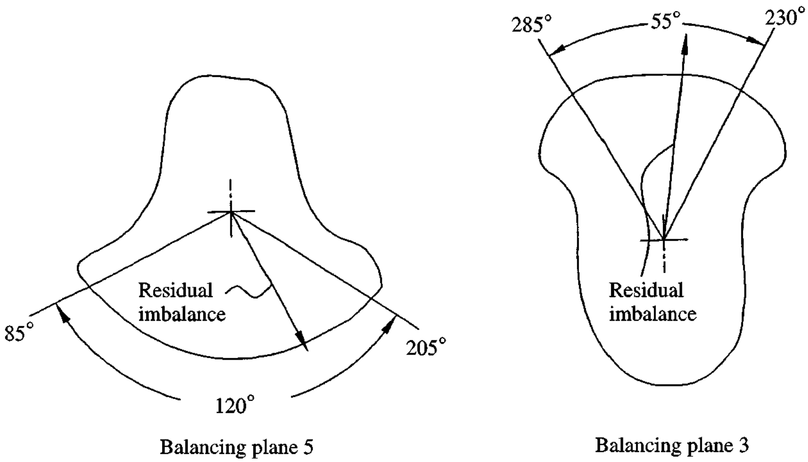


Figure 10. The final residual imbalance vector in the second case.

Table 6

*The comparison of correction between two methods in the second case*

| No. of balancing planes | The modified method: Location angle (deg) | The modified method: Imbalance (g mm) | The conventional method: Location angle (deg) | The conventional method: Imbalance (g mm) |
|---|---|---|---|---|
| 1 | 60 | 0 | 60 | 0 |
| | 300 | 1720·9 | 300 | 1261·2 |
| 2 | 20 | 0 | 20 | 0 |
| | 330 | 0 | 330 | 0 |
| 3 | 230 | 945·4 | 230 | 1331·8 |
| | 285 | 41·6 | 285 | 536·3 |
| 4 | 130 | 0 | 130 | 0 |
| | 180 | 0 | 180 | 0 |
| 5 | 85 | 1003·5 | 85 | 1771·1 |
| | 205 | 835 | 205 | 473·4 |
| Total amounts | — | 4544·9 | — | 5373·8 |

The comparisons of the amounts of correction mass and the final residual responses between the modified method and conventional method are shown in Figure 11. Conventional and modified methods involve the computation of the influence coefficient and the determination of imbalance by using equation (21), the required time for one computation being below 1 s. Two and four iterations of the modified method for four- and six-cylinder crank-shafts, respectively, give these final results. However, the determination of composition needs additional computation time, which is longer than the time needed for the modified method.

For the measurement noise and the machine inaccuracy, final residual responses due to both methods cannot be zero. Consequently, both cases indicate that the modified method brings about better quality as compared with the conventional method for balancing crank-shafts.

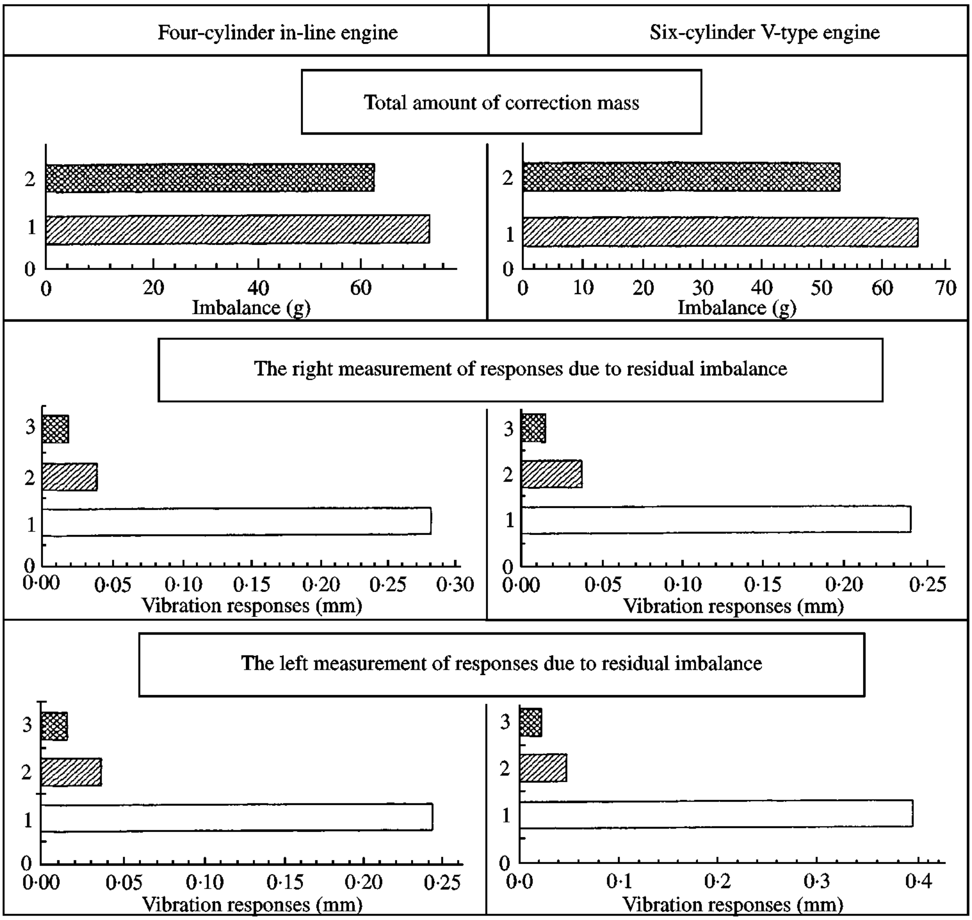


Figure 11. A comparison between the modified and the conventional methods: □, of the original responses; ▩, of the modified method; ▨, of the conventional method.

## 9. CONCLUSIONS

This study has demonstrated how the soft-pedestal machine can be utilized for balancing engine crank-shafts. A modified approach, which is verified on the basis of the modified influence coefficient theory and the iteration correction technique, has been presented. A higher balancing quality can be achieved by the modified method, which has been verified by experiments in case studies.

## ACKNOWLEDGMENT

This study was supported by the National Science Council of the Republic of China under grant no. NSC 86-2612-E033-001.

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## APPENDIX A. NOMENCLATURE

| | |
|---|---|
| $A_{kj}$, $B_{kj}$ | element in the $k$th row and the $j$th column of matrices $[A]$, $[B]$ |
| $[A]$, $[B]$ | influence coefficient matrices |
| $[C]$ | assembly of influence coefficient matrices $[A]$ and $[B]$ in real form |
| $[D]$ | the half of influence coefficient matrices $[C]$ |
| $f$ | amplitude of forward precession |
| $f_r$ | response due to residual imbalance |
| $i$ | $\sqrt{-1}$ |
| $J$, $K$ | number of balancing planes, measuring points |
| $P_j$, $Q_j$ | imbalance force |
| $q$ | synchronous whirl |
| $q^+$, $q^-$ | relative components of forward precession, backward precession to a rotating reference respectively |
| $r$ | translations in a complex form |
| $[W]$ | diagonal matrix is composed of the weighting factors |
| $\mu$ | imbalance |
| $\mu_r$ | residual imbalance |
| $\theta$ | an included angle of $\mu$ and 0-co-ordinate axis |
| $\psi$ | phase measured from a rotating reference |
| $\Omega$ | rotating speed |
| $\phi$ | included angle for the limits of removable masses on counterbalance |

*Superscript*

| | |
|---|---|
| $I$ | imaginary part of a complex variable |
| $o$ | original imbalance |
| $R$ | real part of a complex variable, right correction plane |
| $T$ | transpose |
| $'$ | trial mass at the first time/residual imbalance at the first iteration |
| $''$ | trial mass at the second time/residual imbalance at the second iteration |
| $0$, $\phi$ | 0-axis and $\phi$-axis of $\phi$-co-ordinates |

*Subscript*

| | |
|---|---|
| $c$, $s$ | cosine term, sine term |
| $i$ | the equivalent imbalance |
| $j$, $k$ | the $j$th balancing plane, the $k$th measurement point |
| $L$ | left correction plane |
| $M$ | median correction plane |
| $n$ | the number of combinations of any two counterbalance planes |
| $R$ | right correction plane |
| $v$, $w$ | components in $V$, $W$ directions |